



TABELAS DE ANÁLISE MATEMÁTICA

Tabela de Trigonometria

$\operatorname{tg}(x) = \frac{\operatorname{sen}(x)}{\operatorname{cos}(x)}$	$\operatorname{cotg}(x) = \frac{1}{\operatorname{tg}(x)}$	$\operatorname{sec}(x) = \frac{1}{\operatorname{cos}(x)}$	$\operatorname{cosec}(x) = \frac{1}{\operatorname{sen}(x)}$
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1. $\operatorname{sen}^2(x) + \operatorname{cos}^2(x) = 1$	8. $\operatorname{cos}(2x) = \operatorname{cos}^2(x) - \operatorname{sen}^2(x)$
2. $1 + \operatorname{tg}^2(x) = \operatorname{sec}^2(x)$	9. $\operatorname{tg}(2x) = \frac{2\operatorname{tg}(x)}{1 - \operatorname{tg}^2(x)}$
3. $1 + \operatorname{cotg}^2(x) = \operatorname{cosec}^2(x)$	10. $\operatorname{sen}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \operatorname{cos}(x)}{2}}$
4. $\operatorname{sen}(x \pm y) = \operatorname{sen}(x)\operatorname{cos}(y) \pm \operatorname{sen}(y)\operatorname{cos}(x)$	11. $\operatorname{cos}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 + \operatorname{cos}(x)}{2}}$
5. $\operatorname{cos}(x \pm y) = \operatorname{cos}(x)\operatorname{cos}(y) \mp \operatorname{sen}(x)\operatorname{sen}(y)$	12. $\operatorname{tg}\left(\frac{x}{2}\right) = \pm \sqrt{\frac{1 - \operatorname{cos}(x)}{1 + \operatorname{cos}(x)}}$
6. $\operatorname{tg}(x \pm y) = \frac{\operatorname{tg}(x) \pm \operatorname{tg}(y)}{1 \mp \operatorname{tg}(x)\operatorname{tg}(y)}$	13. $\operatorname{sen}(x) = \frac{2\operatorname{tg}(x/2)}{1 + \operatorname{tg}^2(x/2)}$
7. $\operatorname{sen}(2x) = 2\operatorname{sen}(x)\operatorname{cos}(x)$	14. $\operatorname{cos}(x) = \frac{1 - \operatorname{tg}^2(x/2)}{1 + \operatorname{tg}^2(x/2)}$

15. $\operatorname{sen}(x) \pm \operatorname{sen}(y) = 2\operatorname{sen}\left(\frac{x \pm y}{2}\right)\operatorname{cos}\left(\frac{x \mp y}{2}\right)$
16. $\operatorname{cos}(x) + \operatorname{cos}(y) = 2\operatorname{cos}\left(\frac{x + y}{2}\right)\operatorname{cos}\left(\frac{x - y}{2}\right)$
17. $\operatorname{cos}(x) - \operatorname{cos}(y) = -2\operatorname{sen}\left(\frac{x + y}{2}\right)\operatorname{sen}\left(\frac{x - y}{2}\right)$

	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$	π	$3\pi/2$
Seno	0	1/2	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0	-1
Coseno	1	$\sqrt{3}/2$	$\sqrt{2}/2$	1/2	0	-1	0
Tangente	0	$\sqrt{3}/3$	1	$\sqrt{3}$	-	0	-

Tabela de Derivadas

$u = f(x)$	$v = g(x)$	$\alpha = \text{constante}$	$a = \text{constante}$	$k = \text{constante}$
1. $k' = 0$				17. $(\operatorname{tg} u)' = u' \sec^2 u$
2. $x' = 1$				18. $(\operatorname{cotg} u)' = -u' \operatorname{cosec}^2 u$
3. $(u \pm v)' = u' \pm v'$				19. $(\sec u)' = u' \sec u \operatorname{tg} u$
4. $(uv)' = u'v + uv'$				20. $(\operatorname{cosec} u)' = -u' \operatorname{cosec} u \operatorname{cotg} u$
5. $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}$				21. $(\operatorname{arc} \operatorname{sen} u)' = \frac{u'}{\sqrt{1-u^2}}$
6. $(ku)' = ku'$				22. $(\operatorname{arc} \operatorname{cos} u)' = -\frac{u'}{\sqrt{1-u^2}}$
7. $(u^\alpha)' = \alpha u^{\alpha-1} u'$				23. $(\operatorname{arc} \operatorname{tg} u)' = \frac{u'}{1+u^2}$
8. $(\sqrt{u})' = \frac{u'}{2\sqrt{u}}$				24. $(\operatorname{arc} \operatorname{cotg} u)' = -\frac{u'}{1+u^2}$
9. $(\sqrt[n]{u})' = \frac{u'}{n\sqrt[n]{u^{n-1}}}$				25. $(\operatorname{arc} \operatorname{sec} u)' = \frac{u'}{u\sqrt{u^2-1}}$
10. $(e^u)' = e^u u'$				26. $(\operatorname{arc} \operatorname{cosec} u)' = -\frac{u'}{u\sqrt{u^2-1}}$
11. $(a^u)' = a^u u' \ln a$				27. $(\operatorname{sh} u)' = u' \operatorname{ch} u$
12. $(u^v)' = u^v v' \ln u + vu^{v-1} u'$				28. $(\operatorname{ch} u)' = u' \operatorname{sh} u$
13. $(\ln u)' = \frac{u'}{u}$				29. $(\operatorname{tgh} u)' = \frac{u'}{\operatorname{ch}^2 u} = u' \operatorname{sech}^2 u$
14. $(\log_a u)' = \frac{u'}{u \ln a}$				30. $(\operatorname{arg} \operatorname{sh} u)' = \frac{u'}{\sqrt{1+u^2}}$
15. $(\operatorname{sen} u)' = u' \operatorname{cos} u$				31. $(\operatorname{arg} \operatorname{ch} u)' = \frac{u'}{\sqrt{u^2-1}}$
16. $(\operatorname{cos} u)' = -u' \operatorname{sen} u$				32. $(\operatorname{arg} \operatorname{tgh} u)' = \frac{u'}{1-u^2}$

Tabela de Primitivas

$$P f(x) = F(x) \Rightarrow F'(x) = f(x)$$

$u = f(x)$	$v = g(x)$	$\alpha = \text{constante}$	$a = \text{constante}$	$k = \text{constante}$
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I - Primitivas Imediatas

1. $P k = kx$	13. $P u' \operatorname{cosec} u \cotg u = -\operatorname{cosec} u$
2. $P ku = kPu$	14. $P \frac{u'}{\sqrt{1-u^2}} = \operatorname{arc} \operatorname{sen} u = -\operatorname{arc} \operatorname{cos} u$
3. $P u^\alpha u' = \frac{u^{\alpha+1}}{\alpha+1}, \alpha \neq -1$	15. $P \frac{u'}{\sqrt{a^2-u^2}} = \operatorname{arc} \operatorname{sen} \frac{u}{a} = -\operatorname{arc} \operatorname{cos} \frac{u}{a}$
4. $P \frac{u'}{u} = \ln u $	16. $P \frac{u'}{1+u^2} = \operatorname{arc} \operatorname{tg} u = -\operatorname{arc} \operatorname{cotg} u$
5. $P \frac{u'}{2\sqrt{u}} = \sqrt{u}$	17. $P \frac{u'}{a^2+u^2} = \frac{1}{a} \operatorname{arc} \operatorname{tg} \frac{u}{a} = -\frac{1}{a} \operatorname{arc} \operatorname{cotg} \frac{u}{a}$
6. $P e^u u' = e^u$	18. $P \frac{u'}{u\sqrt{u^2-1}} = \operatorname{arc} \operatorname{sec} u = -\operatorname{arc} \operatorname{cosec} u$
7. $P a^u u' = \frac{a^u}{\ln a}$	19. $P u' \operatorname{ch} u = \operatorname{sh} u$
8. $P u' \operatorname{sen} u = -\operatorname{cos} u$	20. $P u' \operatorname{sh} u = \operatorname{ch} u$
9. $P u' \operatorname{cos} u = \operatorname{sen} u$	21. $P u' \operatorname{sech}^2 u = \operatorname{tgh} u$
10. $P u' \operatorname{sec}^2 u = \operatorname{tg} u$	22. $P \frac{u'}{\sqrt{u^2+1}} = \operatorname{arg} \operatorname{sh} u$
11. $P u' \operatorname{cosec}^2 u = -\operatorname{cotg} u$	23. $P \frac{u'}{\sqrt{u^2-1}} = \operatorname{arg} \operatorname{ch} u$
12. $P u' \operatorname{sec} u \operatorname{tg} u = \operatorname{sec} u$	24. $P \frac{u'}{1-u^2} = \operatorname{arg} \operatorname{tgh} u, u < 1$

II - Integração por Partes

$$P(uv) = (Pu)v - P((Pu)v')$$

III - Integração por Substituição

FR (...) indica que se trata de uma fracção que envolve apenas somas, diferenças, produtos e quocientes do que se encontra entre parêntesis.

Função a Primitivar :

1. $\frac{1}{(x^2 + a^2)^k}$, $k \in \mathbb{N}, k > 1$

2. $\frac{P(x)}{(ax^2 + bx + c)^k}$, $k \in \mathbb{N}, k > 1, b^2 - 4ac < 0$,

onde $P(x)$ é um polinómio de grau inferior a $2k$

3. $\frac{P(x)}{((x-p)^2 + q^2)^k}$, $k \in \mathbb{N}$,

onde $P(x)$ é um polinómio de grau inferior a $2k$

4. $\frac{x^{k-1}}{x^{2k} + a}$, $k \in \mathbb{Q}$

5. FR (a^{rx} , a^{sx} , ...)

6. FR ($x^{\frac{p}{q}}$, $x^{\frac{r}{s}}$, ...)

7. FR ($x, (ax+b)^{\frac{p}{q}}, (ax+b)^{\frac{r}{s}}$, ...)

8. FR ($x, (\frac{ax+b}{cx+d})^{\frac{p}{q}}, (\frac{ax+b}{cx+d})^{\frac{r}{s}}$, ...)

9. FR ($x, \sqrt{a^2 - b^2 x^2}$)

10. FR ($x, \sqrt{a^2 + b^2 x^2}$)

11. FR ($x, \sqrt{b^2 x^2 - a^2}$)

12. FR ($x, \sqrt{x}, \sqrt{a - bx}$)

13. FR ($x, \sqrt{x}, \sqrt{a + bx}$)

Substituição a Efectuar :

1. $x = a \operatorname{tg}(t)$

2. $ax + \frac{b}{2} = t$

3. $x = p + qt$

4. $x^k = at$

5. $a^{mx} = t$, $m = \text{m.d.c.}(r, s, \dots)$

6. $x = t^m$, $m = \text{m.m.c.}(q, s, \dots)$

7. $ax + b = t^m$, $m = \text{m.m.c.}(q, s, \dots)$

8. $\frac{ax+b}{cx+d} = t^m$, $m = \text{m.m.c.}(q, s, \dots)$

9. $x = \frac{a}{b} \operatorname{sen}(t)$ ou $x = \frac{a}{b} \operatorname{cos}(t)$

10. $x = \frac{a}{b} \operatorname{tg}(t)$

11. $x = \frac{a}{b} \operatorname{sec}(t)$

12. $x = \frac{a}{b} \operatorname{sen}^2(t)$ ou $x = \frac{a}{b} \operatorname{cos}^2(t)$

13. $x = \frac{a}{b} \operatorname{tg}^2 t$

14. $\text{FR}(x, \sqrt{x}, \sqrt{bx-a})$

14. $x = \frac{a}{b} \sec^2(t)$

15. $\text{FR}(x, \sqrt{ax^2 + bx + c})$

15. Se $a > 0$ faz-se $\sqrt{ax^2 + bx + c} = x\sqrt{a} + t$

Se $c > 0$ faz-se $\sqrt{ax^2 + bx + c} = \sqrt{c} + tx$

Noutros casos, bem como nos anteriores,

faz-se $\sqrt{ax^2 + bx + c} = (x - \alpha)t$ ou

$\sqrt{ax^2 + bx + c} = (x - \beta)t$, com

$ax^2 + bx + c = a(x - \alpha)(x - \beta)$

16. $x^m (a + bx^n)^{\frac{p}{q}}$

16. Se $\frac{m+1}{n} \in \mathbb{Z}$ faz-se $a + bx^n = t^q$

Se $\frac{m+1}{n} + \frac{p}{q} \in \mathbb{Z}$ faz-se $a + bx^n = x^n t^q$

17. $\text{FR}(\text{sen}(mx), \text{cos}(mx))$

17. $mx = t$

18. $\text{FR}(\text{sen}(x), \text{cos}(x))$

18.

a) Se FR é ímpar em $\text{cos}(x)$,

a) $\text{sen}(x) = t$

isto é, se:

$\text{FR}(\text{sen}(x), -\text{cos}(x)) = -\text{FR}(\text{sen}(x), \text{cos}(x))$

b) Se FR é ímpar em $\text{sen}(x)$,

b) $\text{cos}(x) = t$

isto é, se:

$\text{FR}(-\text{sen}(x), \text{cos}(x)) = -\text{FR}(\text{sen}(x), \text{cos}(x))$

c) Se FR é par em $\text{sen}(x)$

c) $\text{tg}(x) = t$ (supondo $x \in \left]0, \frac{\pi}{2}\right[$), sendo

e $\text{cos}(x)$, isto é, se

$\text{sen}(x) = \frac{t}{\sqrt{1+t^2}}$ e $\text{cos}(x) = \frac{t}{\sqrt{1+t^2}}$

$\text{FR}(-\text{sen}(x), -\text{cos}(x)) = \text{FR}(\text{sen}(x), \text{cos}(x))$

d) Nos restantes casos,

d) $\text{tg}\left(\frac{x}{2}\right) = t$, sendo $\text{sen}(x) = \frac{2t}{1+t^2}$ e

assim como nos anteriores

$\text{cos}(x) = \frac{1-t^2}{1+t^2}$